Stochastic Lagrangian Trajectory Modeling of Surface Drifters Deployed during the Deepwater Horizon Oil Spill

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Abstract

Lagrangian trajectory models have been demonstrated to be a useful tool in oil spill response. Despite the improvements in this kind of models, surface drift prediction remains a difficult task plagued with uncertainties. This work presents a Stochastic Lagrangian Trajectory Model (SLTM) that quantifies the uncertainties in trajectory simulations and defines the most likely search area of possible trajectories. The methodology includes the following steps: 1) Numerical scheme based on the transport equation in Lagrangian form; (2) Parameter estimation process, which includes: (i) time independent parameter estimation based on the maximum likelihood method and (ii) time dependent parameter estimation through autoregressive moving average; (3) Monte Carlo simulation of multiple trajectories based on the joint probability distribution function and the temporal dependency model. The model is used to simulate the trajectories of surface drifters deployed in the Gulf of Mexico during the *Deepwater Horizon* oil spill incident using surface currents provided by the Global model HYCOM. A set of drifters was selected to estimate the model parameters and another one for simulation and validation. Observed drifter trajectories were compared with the modeled trajectories obtained with the deterministic and the SLTM approaches. After 5 days of simulation the root mean squared error of Lagrangian separation distance was found to be 115 km and 37 km for the deterministic approach and the best simulation of the SLTM, respectively. Moreover, actual trajectories were in the areas where the model predicted that the drifter was likely to go through, showing the capabilities of the SLTM for oil spill trajectory modelling and search and rescue applications.

1 Introduction

The prediction of the trajectory of oil spills in the ocean is of great importance in an oil spill response. Recent oil spill accidents, as the *Prestige* accident is Spain (2002) and the case of the *Deepwater Horizon* oil spill in the Gulf of Mexico in 2010, have shown that forecasting the

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oil spill trajectory is fundamental for planning mitigation strategies (Castanedo et al., 2006; Liu et al., 2011c). Lagrangian trajectory models based on hydrodynamic and atmospheric models have been widely used to predict the trajectory of oil spills (e.g., Spaulding et al., 1992; Beegle-Krause, 2001; Daniel et al., 2003; Castanedo et al., 2006; Abascal et al., 2009a).

Despite the advances and improvements in this kind of models, surface drift prediction in the ocean is a difficult task because of the complexity of all processes involved, uncertainties in drift properties and in the environmental conditions. Therefore, the accuracy of the oil spill trajectory forecast depends on the formulation of the Lagrangian model itself and also on the accuracy of the met-ocean forcing data, usually provided by hydrodynamic and meteorological numerical models. All forecast models have errors that grow with time, which can lead to significant errors in the trajectory prediction. Given the relevance of the uncertainty in the trajectory forecast, several studies have been carried out recently to take into account the different sources of uncertainty in the trajectory simulations (e.g., Sebastião and Soares, 2006; Abascal et al., 2009b; Rixen et al., 2008; Liu and Weisberg, 2011).

The uncertainty can be accounted for in a stochastic framework (Mínguez et al., 2012). In this approach, the relevant parameters are considered as random variables and their uncertainty is quantified in terms of a probability distribution function. Once the random variables are determined, a Monte Carlo method (Rubinstein and Kroese, 2011) can be applied to simulate multiple trajectories and establish a search area for the location of the oil spill (or drifter objects). The advantage of dealing with uncertainty is that simple models, representing the most important physical processes, may be used instead of complex models. In these cases, the appropriate definition of the uncertainty is more important than the complexity of the model itself.

In this work, the application of a Stochastic Lagrangian Trajectory Model (SLTM) (Mínguez et al., 2012) for oil spill modelling is presented. This model is able to quantify the uncertainties in trajectory simulations and defines the most likely search area for possible trajectories. The model is used to simulate the trajectories of surface drifters deployed in the Gulf of Mexico during the *Deepwater Horizon* oil spill incident (Liu and Weisberg, 2011) using surface currents provided by the Global HYbrid Coordinate Ocean Model (HYCOM) (e.g., Bleck, 2002; Chassignet et al., 2003). A set of drifters is used for the parameter estimation process and another one for simulation and validation. Taking into account that the HYCOM is traditionally a deep ocean application model (Chassignet et al., 2003; Shaji et al., 2005), this work focuses on the simulation of the drifter trajectories located in deep ocean. The validation is performed by means of the comparison of the search area calculated with the SLTM and the observed drifter trajectories. Moreover, comparisons with the deterministic approach are performed in order to analyze the advantages of the SLTM.

This paper is structured as follows. Next section describes the data used in this study. Following, the methodology and results are presented. Finally, the main conclusions are summarized.

2 Data

Drifters used in this work were deployed by the Ocean Circulation Group (OCG) within the USF College of Marine Science in response to the Deepwater Horizon oil spill (Liu et al., 2011a). Drifters were deployed in May 2010 in the Loop Current, its shed eddy and on the West Florida Shelf to help monitor the evolution of the regional flow fields (Liu et al., 2011b). Such information further served in assessing the trajectories as estimated by the models employed to track the spilled oil (e.g., http://ocgweb.marine.usf.edu). Six drifters were initially deployed during a 19–24 May 2010 R/V Bellows cruise joint between the USF OCG, the USF Optical

Oceanography Laboratory, the Florida Department of Environmental Protection (FDEP), the U.S. Coast Guard (USCG), and Florida Wildlife Research Institute (FWRI). Three drifters were subsequently deployed during a 2–14 June 2010 R/V Weatherbird II cruise by the USF OCG assisted by the Florida Institute of Technology (FIT). Nine more drifters were then added during a 22–25 June 2010 R/V Weatherbird II cruise, in a joint effort by the USF OCG, the Woods Hole Oceanographic Institution (WHOI), and the Northeast Fisheries Science Center (NEFSC). The drifters, drogued at 1 m depth, transmitting data via satellite in real time. The locations of the drifter trajectories were binned at hourly time steps and archived. Figure 1 shows the trajectories for May–August 2010. Note that the paths in the open ocean (outside the continental slope) are represented by the thinner lines. These drifter data have been used to describe the ocean circulation patterns in the eastern Gulf of Mexico, and to assess the performance of trajectory models (Liu and Weisberg, 2011) and altimetry products (Liu et al., 2014).

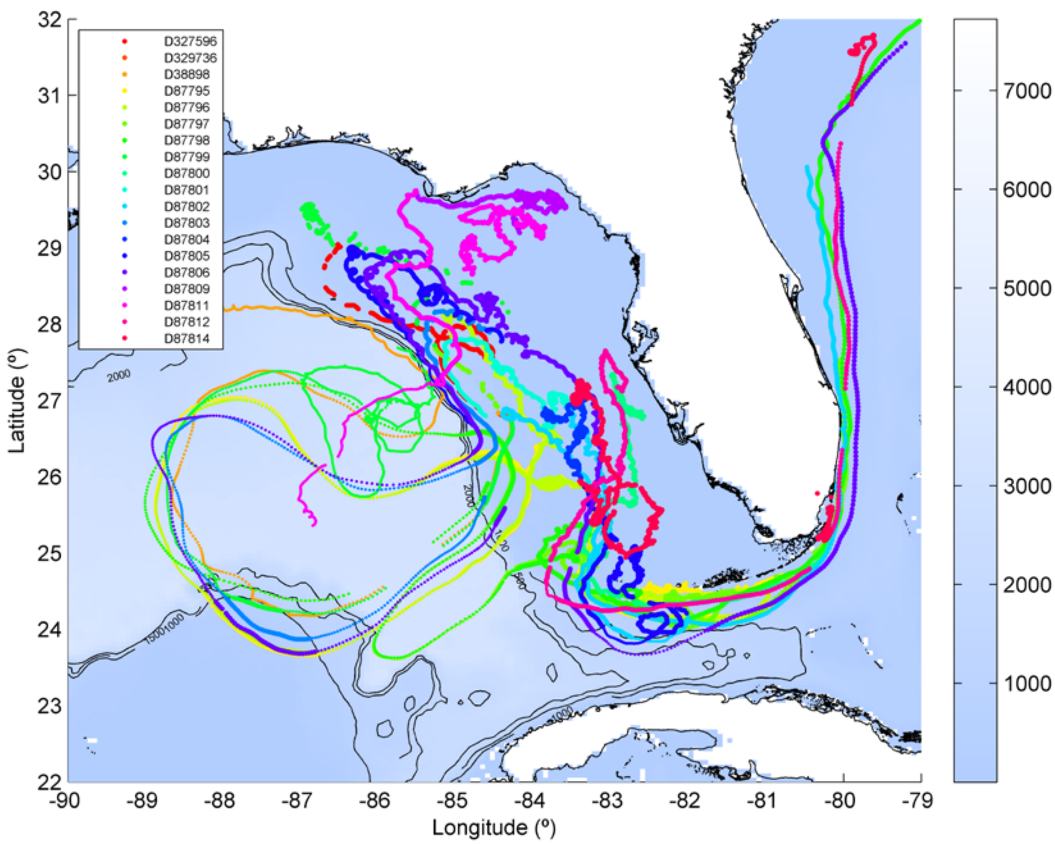

Figure 1 Drifters released in the eastern Gulf of Mexico during May-August 2010. The thinner lines represent the paths in the open ocean.

Ocean currents were obtained with the Global HYCOM(e.g., Bleck, 2002; Chassignet et al., 2003), which is configured to simulate global ocean circulation on a Mercator grid with 1/12° equatorial resolution (e.g., Chassignet et al., 2007, 2009). The horizontal resolution in the Gulf of Mexico is about 9 km. Surface forcing is from Navy Operational Global Atmospheric Prediction System (NOGAPS) (Hogan and Rosmond, 1991; Rosmond, 1992) and includes wind stress, wind speed, heat flux (using bulk formula), and precipitation. Data assimilation is via the Navy Coupled Ocean Data Assimilation (NCODA) system (Cummings, 2005), which uses the

Modular Ocean Data Assimilation System (MODAS) synthetic data product (Fox et al., 2002). The Global HYCOM and NCODA hindcast experiment output are available as daily snapshots via the HYCOM Consortium website (http://www.hycom.org/). This study uses the surface velocity fields that where interpolated into 3-hourly time series (as in the work of Liu and Weisberg, 2011).

3 Methodology

The Stochastic Lagrangian trajectory model (SLTM) used in this work is a particular application for oil spills of the SLTM for drifting objects developed by Mínguez et al. (2012). The model encompasses the following stages: 1) Numerical scheme based on the transport equation in Lagrangian form; (2) Parameter estimation process, which includes: (i) time independent parameter estimation based on the maximum likelihood method and (ii) time dependent parameter estimation through autoregressive moving average; (3) Monte Carlo simulation of multiple trajectories based on the joint probability distribution function and the temporal dependency model.

3.1 Lagrangian Trajectory Model

Oil spills on the sea surface are transported by the combined effect of surface ocean currents, wind, waves and turbulent dispersion. This transport is governed by the transport equation in Lagrangian form:

$$\frac{d\vec{x}}{dt} = \vec{U}_a\left(\vec{x}_i, t\right) + \vec{U}_d\left(\vec{x}_i, t\right) \tag{1}$$

where $\vec{x}$ is the position and $\vec{U}_a$ and $\vec{U}_d$ are the advective and the diffusive velocities, respectively. Assuming a partial transference of momentum from wind and waves, the advective velocity of the oil slick, $\vec{U}_a$, can be expressed as

$$\vec{U}_a = \vec{U}_C + C_D \vec{U}_W + C_H \vec{U}_H \tag{2}$$

where $\vec{U}_C$ is the surface current velocity, $\vec{U}_W$ is the wind velocity at a height of 10 meters over the sea surface, $\vec{U}_H$ is the wave-induced Stokes drift, C_D is the wind drag coefficient and C_H is the wave coefficient. Note that traditionally, the advection term is assumed to be deterministic, where the optimal parameters C_D and C_H are between 3-3.5% and 0.01-1.5% (ASCE, 1996; Castanedo et al., 2006; Abascal et al., 2009b). However, in this model coefficients C_D and C_H are assumed to be normally distributed random variables with parameters $(\mu_{C_D}, \sigma^2_{C_D})$ and $(\mu_{C_H}, \sigma^2_{C_H})$, respectively.

The diffusive velocity depends on the sea turbulence characteristics and it is assumed to be stochastic. Assuming that the angle between the advection velocity and the abscissas axis is γ, the velocity $\vec{U}_d$ is given by:

$$\vec{U}_d = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) \\ \sin(\gamma) & \cos(\gamma) \end{pmatrix} \begin{pmatrix} D_L \\ D_T \end{pmatrix} \tag{3}$$

where the longitudinal (D_L) and transversal (D_T) diffusive components are normally distributed random variables with parameters $(\mu_{D_L}, \sigma^2_{D_L})$ and $(\mu_{D_T}, \sigma^2_{D_T})$ respectively.

Note that a general model is presented. However, the final model will depend upon data availability for each particular application. In this study, only surface currents data are considered.

3.1.1 Numerical Scheme

Solving Eq. (1) by means of the first-order Euler method and taking into account that only surface currents data are considered in this study, the oil spill location at every time step is defined as:

$$\vec{x}_t = \vec{x}_{t-1} + \Delta t\left[\vec{U}_a\left(\vec{x}_{t-1}\right) + \vec{U}_d\left(\vec{x}_{t-1}\right)\right] = \vec{x}_{t-1} + \Delta t\left[\vec{U}_C\left(\vec{x}_{t-1}\right) + \vec{U}_d\left(\vec{x}_{t-1}\right)\right] \quad (4)$$
$$t = 1, \dots, n_{t;}$$

where $\vec{x}_t$ is the oil spill location at time *t*, $\vec{U}_a(\vec{x}_t)$, $\vec{U}_d(\vec{x}_t)$ and $\vec{U}_C(\vec{x}_t)$ are, respectively, the advection, the diffusion and the surface current at the oil spill location at time *t* and n_t is the number of time intervals considered.

Using (3), the diffusive velocity can be expressed as:

$$\vec{U}_d(\vec{x}_{t-1}) = \begin{pmatrix} \cos(\gamma_{t-1}) & -\sin(\gamma_{t-1}) \\ \sin(\gamma_{t-1}) & \cos(\gamma_{t-1}) \end{pmatrix} \begin{pmatrix} D_L^{t-1} \\ D_T^{t-1} \end{pmatrix} \quad (5)$$

where γ_{t-1} is the direction angle of the advection velocity $\vec{U}_a(\vec{x}_{t-1})$, and D_L^t and D_T^t are the longitudinal and transversal diffusive velocities at the oil spill location at time *t*. As mentioned, random model variables within vector $\boldsymbol{p} = \left(D_L, D_T\right)^t$ are assumed to follow a normal distribution with their corresponding mean and standard deviation parameters. It is worth mentioning that random model parameters take into account, implicitly, all the uncertainties involved in the simulation process (e.g., uncertainty in the forcing data, numerical scheme). As mentioned above, only currents data have been used in this study. Thus, the only random variables considered are the transversal (D_T) and longitudinal (D_L) diffusive components. Note that the diffusive components are taking into account all the uncertainties involved in the simulation process and they do not represent an estimate of physical dispersive processes.

3.2 Parameter Estimation

The parameter estimation process is based on two different stages: (1) time independent parameter estimation based on the maximum likelihood method and (2) time dependent parameter estimation through autoregressive moving average. These processes are described below.

3.2.1 The Maximum Likelihood Method

The first step of the parameter estimation process consists in estimating the parameters of the transversal (D_T) and longitudinal (D_L) random diffusive components by means of the maximum likelihood method. The maximum likelihood method is based on maximizing the likelihood of an observed sample, and it can be used to derive point and interval parameter estimates. In this study, the observed sample corresponds to the drifter trajectories previously described. We assume that the random probability distribution parameters are estimated so that, using Eq. 4, the likelihood of the model to reproduce the given trajectories is maximized.

Following the methodology proposed by Mínguez et al. (2012), the mean and standard deviation parameters $\theta = (\mu_{D_L}, \sigma^2_{D_L}, \mu_{D_T}, \sigma^2_{D_T})$ can be estimated using the log-likelihood function by solving the following optimization problem:

$$\underset{\theta, p_k^{i,t}}{Maximize} \sum_{i=1}^{n_p} \sum_{t=1}^{n_t} \sum_{i=1}^{n_d} \log\left[f_k\left(p_k^{i,t}; \theta \right) \right] \tag{6}$$

where $p_k^{i,t}$ corresponds to the actual value of random variable k from the vector $\boldsymbol{p}$ for drifter trajectory i at time t, n_p is the number of random variables considered, n_t is the number of time intervals, n_d is the number or drifting buoys, and subject to the following constraints:

$$\vec{x}_t^{\,i} - \vec{x}_{t-1}^{\,i} = \Delta t \left[\vec{U}_a\left(\vec{x}_{t\text{-}1}^{\,i} \right) + \vec{U}_d\left(\vec{x}_{t\text{-}1}^{\,i} \right) \right]; \quad t = 1, \ldots, n_{t;} \quad i = 1, \ldots, n_d; \tag{7}$$

$$\vec{U}_a(x_{t-1}^i) = \vec{U}_C(\vec{x}_{t-1}^{\,i}); \quad \forall t; \forall i \tag{8}$$

$$\gamma_{t-1}^i = \arctan\left(\frac{U_{a_y}^i}{U_{a_x}^i} \right); \quad t = 1, \ldots, n_t; \quad i = 1, \ldots, n_d; \tag{9}$$

$$\vec{U}_d(\vec{x}_{t-1}^{\,i}) = \begin{pmatrix} \cos(\gamma_{t-1}^i) & -\sin(\gamma_{t-1}^i) \\ \sin(\gamma_{t-1}^i) & \cos(\gamma_{t-1}^i) \end{pmatrix} \begin{pmatrix} D_L^{i,t-1} \\ D_T^{i,t-1} \end{pmatrix} \tag{10}$$

$$\sigma_{D_L} > 0; \sigma_{D_T} > 0 \tag{11}$$

where f_k is the probability density function of the random variable k, and $\vec{x}_t^{\,i}$ are the data locations related to the ith drifter trajectory at time t. This problem differs from the traditional maximum likelihood formulation because the actual values of the random variables are unknown and must be obtained from equations (7)-(10). Solving equations (6)-(11) the optimal value ($\hat{\theta}$) of the model parameters is obtained. This optimal value is the MLE (Maximum Likelihood Estimation) of θ. In addition, the most likely values of the random variables ($\hat{p}_k^{i,t}; k = 1, \ldots, n_p; t = 0, \ldots, n_t - 1; i = 0, \ldots, n_d$) are also obtained. These values represent the actual random variable values which allow reproducing the given trajectory with maximum probability.

3.2.2 Temporal Dependence

The parameter estimation method presented in the previous section assumes temporal independence related to the random variables. However, the most likely values of the random variables may hide a temporal dependency structure. This temporal dependency is explored after the parameter estimation by means of ARMA (AutoRegressive Moving Average models) models. In order to fulfill ARMA models requirements, a new stochastic process for each variable, Z, with a standard normal marginal distribution is defined:

$$z = \Phi^{-1}\left[F_Y\left(z\right)\right] \tag{12}$$

where F_Y is the cumulative distribution function (CDF) of the marginal distribution associated with the original stochastic process Y and $\Phi\left(.\right)$ is the CDF of the standard normal random variable.

The stochastic temporal dependence of the random variables is reproduced using transformation (12) and the following univariate ARMA models:

$$z_t^k = \sum_{j=1}^{r^k} \phi_j^k z_{t-j}^k + \varepsilon_t^k - \sum_{j=1}^{q^k} \theta_j^k \varepsilon_{t-j}^k \tag{13}$$

where ε are the residuals, which are uncorrelated $E\left[\varepsilon_t^k \varepsilon_{t-j}^k\right] = 0; \ \ \forall k = 1, ..., n_p$.

Once the model parameters in (13) are estimated, it is possible to reproduce not only the marginal distribution related to the random model variables, but also its temporal dependency. This characteristic is used afterwards for the simulation of new trajectories.

3.3 Monte Carlo Simulation

After the parameters of the model are estimated, a Monte Carlo method in conjunction with the joint probability distribution function and the temporal dependency model presented in the previous section is applied to simulate multiple trajectories. This procedure provides an ensemble of numerical positions that defines the search area of the oil spill location.

Maximum likelihood estimated parameters $\hat{\theta}$, the estimates of the ARMA models defined in (13) and the standard deviation of the corresponding residuals, $\sigma_\varepsilon^k; \ \forall k = 1, ..., n_p$ are used as inputs of the Monte Carlo simulation. Moreover, the initial location of the oil spill $\vec{x}_0$, the Euler time step Δt, the met-ocean forcing data, the number of trajectories n_s and time steps t_s to be simulated are also required.

The Monte Carlo simulation encompasses the following steps: 1) *Error simulation*: simulate the vectors $\tilde{\varepsilon}^k$; 2) *ARMA*: use those simulated errors in the ARMA models given in (13); 3) *Inverse transformatio*n: Get the simulated values of the random model variables using the inverse of transformation (12) and 4) *Trajectory generation*: reproduce the simulated trajectory through the Eq.(4). This procedure is repeated n_s times and as result n_s simulated trajectories are obtained. More details can be found in Mínguez et al. (2012).

4 Results

4.1 Parameter Estimation

As a first step of the methodology the longitudinal (D_L) and transversal (D_T) random diffusive parameters were estimated. The parameter estimation was performed using two drifter trajectories for the period May 25th 2010 at 18:00 to June 8th 2010 at 00:00 (see Figure 2). Daily currents fields were interpolated into 3-hourly time series. To be consistent with currents temporal resolution, 3-hourly drifter data were also considered in this process.

The MLEs and the 95% confidence intervals of the D_T and D_L random variable probability distributions are presented in Table 1. Given that the units are in grades per second, the values have been scaled (multiplied) by 10^5 to highlight the differences. As can be observed, the magnitude of the mean value and the standard deviation are similar for both parameters, indicating similar uncertainty in the longitudinal and transversal component.

The MLE estimation process provides the values of the random variables D_L and D_T which allows reproducing the drifter trajectories. Figure 2 (right panel) shows the comparison between the drifter trajectories, labeled as Data 1 and Data 2 (black and gray solid circle markers) and the trajectories estimated by the maximum likelihood method, labeled as MLE 1 and MLE 2 (blue and red circle markers). As can be observed, simulated trajectories are identical to the true trajectory data.

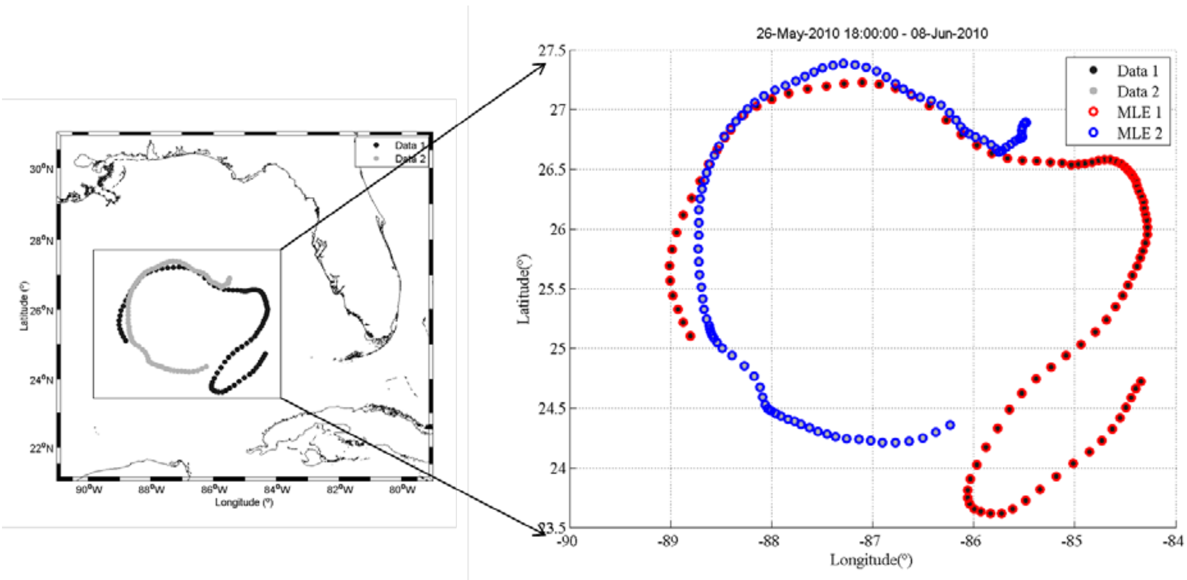

Figure 2 Drifter trajectories selected for the MLE estimation problem (Data 1 and 2), and the final trajectories estimated by the maximum likelihood method (MLE 1 and 2).

Table 1 MLEs and 95% Confidence Intervals of the D_L and D_T Random Variable Probability Distributions

Parameter	Lower bound	Mean	Upper bound
$\mu_{D_T}\ (^\circ/s)$	-0.27142006	-0.19355899	-0.11569793
$\sigma_{D_T}\ (^\circ/s)$	0.49372723	0.54892431	0.60412138
$\mu_{D_L}\ (^\circ/s)$	-0.15356396	-0.10494452	-0.056325076
$\sigma_{D_T}\ (^\circ/s)$	0.30830228	0.34276946	0.37723663

4.2 Temporal Dependence

As a second step of the methodology, the temporal dependence of the stochastic processes related to the variables D_L^t and D_T^t was explored. Figure 3 shows the autocorrelation

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and partial autocorrelation functions related to the maximum likelihood values of the random variables $\hat{Z}_t^{D_T}$ (panels above) and $\hat{Z}_t^{D_L}$ (panels below). Note that the temporal resolution of the data, and therefore, the time lag is 3 hours. The autocorrelation function decays gradually in both processes. Moreover, there are two partial autocorrelation coefficients at lag 1 and 2 outside the 95% confidence intervals. This behavior indicates that the processes are very likely to correspond to moving autoregressive processes of order two. Based on these results, an ARMA (2,0) model were fitted to the time series $\hat{Z}_t^{D_T}$ and $\hat{Z}_t^{D_L}$ obtaining the following parameter estimates: $\hat{\phi}_1^{D_T} = -1.202$, $\hat{\phi}_2^{D_T} = 0.3111$ and $\hat{\phi}_1^{D_L} = -1.264$, $\hat{\phi}_2^{D_L} = 0.4195$. The standard deviation estimates of the residuals are $\hat{\sigma}_\varepsilon^{D_T} = 0.3907$ and $\hat{\sigma}_\varepsilon^{D_L} = 0.4166$, respectively. Finally, a Ljung–Box lack-of-fit hypothesis test considering the null hypothesis that no serial correlation at the lags 1, 2, 3, 4, and 5 exist was applied on the residual samples. The *p* values obtained for a 5% significance level are (0.8205 0.6740 0.7736 0.8456 0.8454) for the transversal residuals, and (0.6164 0.4089 0.4476 0.5709 0.7119) for the longitudinal residuals. Note that since in all cases the *p* values are higher than the significance level 0.05, the null hypothesis is accepted.

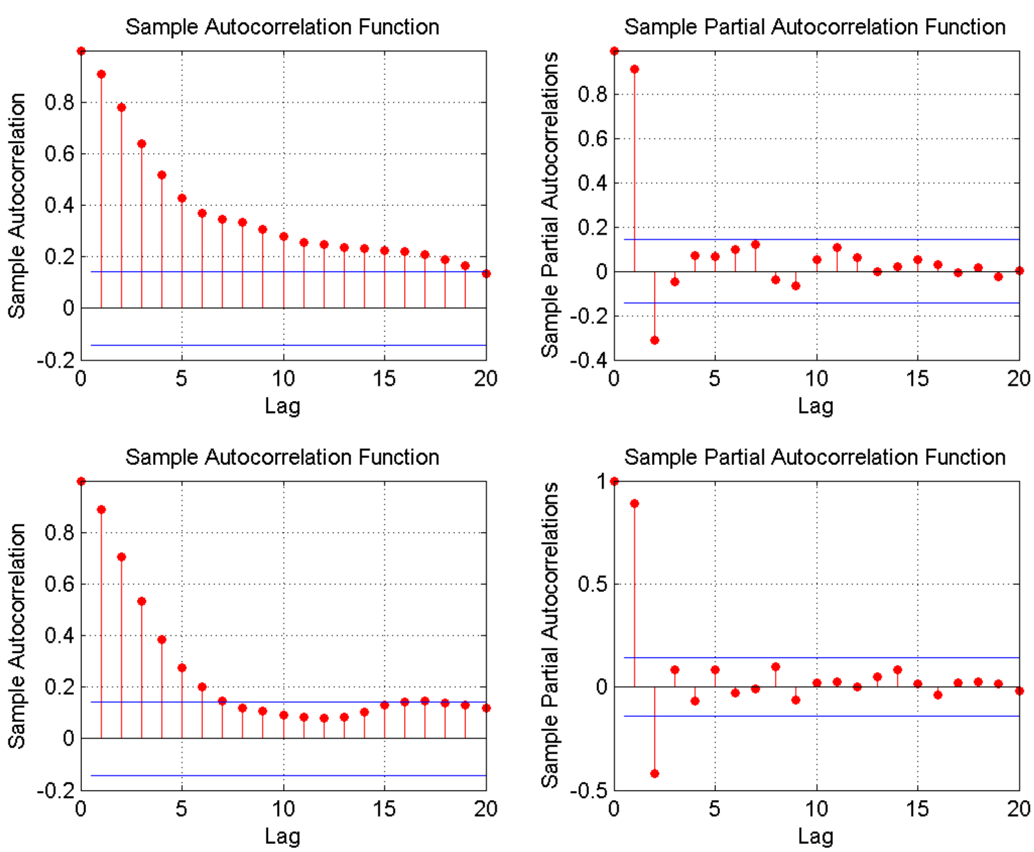

Figure 3 Autocorrelation and partial autocorrelation functions related to the maximum likelihood values of the random variables $\hat{Z}_t^{D_T}$ (upper panels) and $\hat{Z}_t^{D_L}$ (lower panels).

4.3 Simulation and Validation

Once the model parameters were estimated, the next step of the methodology was the simulation of a high number of numerical trajectories (*N*) by means of the Monte Carlo method described in section 3.3. Simulations and validation of the model results were carried out using a different set of drifter trajectories (see Figure 4 and Table 2) during the period May – July 2010. Note that this dataset is independent of the drifter data used for the parameter estimation process. For each drifter trajectory shown in Figure 4 and Table 2, the Lagrangian Stochastic Model was initialized daily from the observed drifter locations at 00:00 h UTC. Subsequently, 10,000 trajectories were simulated with a 5-days forecast horizon period.

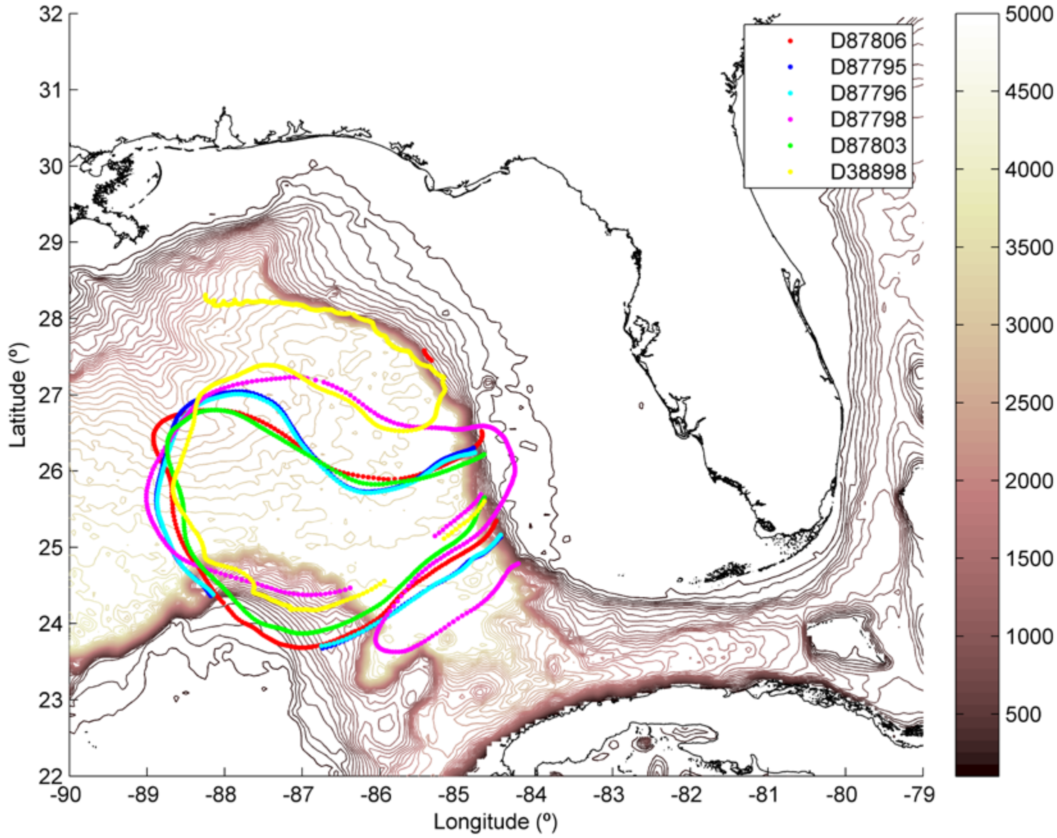

Figure 4 Drifter trajectories used for the validation of the model (May-July 2010). The bathymetry contours are also shown (units in m).

As an example, results of simulations for the drifter # 87806 in the period 06/12/2010 00:00 UTC – 06/21/2010 00:00 UTC (red color in Figure 4) are presented. For each 5-days trajectory section selected, Figure 5 shows the actual drifter trajectory (red circles), the deterministic one (green circles) and the best simulation obtained with the SLTM (black circles), which is the closest to the true ones. In addition, the contour plots related to the density of points for each trajectory is also shown, which represent the search area. The higher the contour value is for each location, the higher the likelihood of the corresponding drifter trajectory to go through the location. As can be observed, actual drifter trajectories are in the areas where the model predicted that drifters were likely to go through.

Table 2 Drifter Trajectories used for the Validation of the Model (May-July 2010)

DRIFTER (#)	INITIAL TIME (UTC)	FINAL TIME (UTC)	Δt (h)
87795	06/14/2010 05:00	06/26/2010 11:00	3
87796	06/14/2010 08:00	06/26/2010 08:00	3
87798	05/25/2010 00:00	06/08/2010 03:00	3
87803	06/10/2010 20:00	06/22/2010 23:00	3
87806	06/11/2010 07:00	06/26/2010 19:00	3
38898	05/26/2010 11:00	06/22/2010 20:00	3

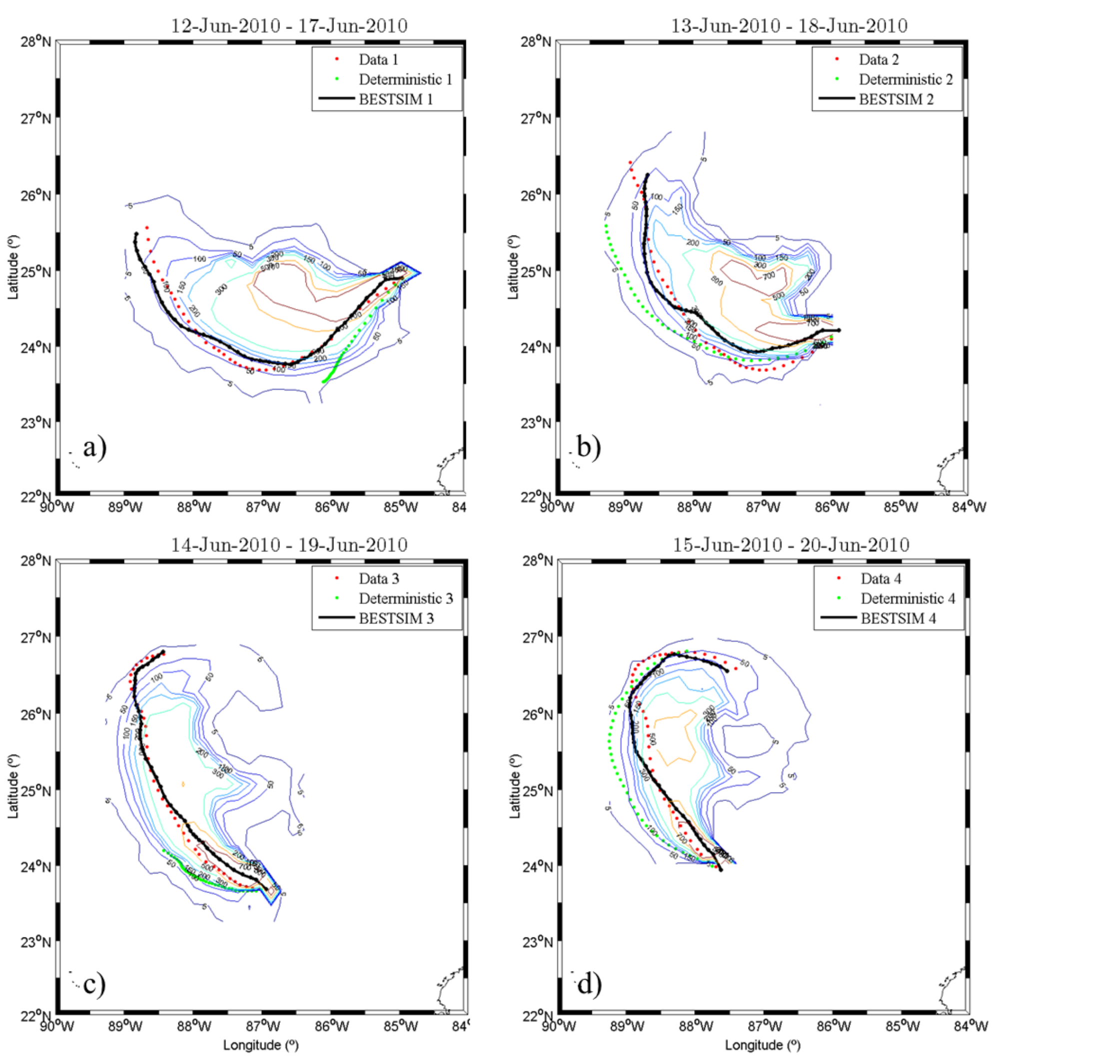

Figure 5 Example of the results for the drifter #87806. The drifter trajectory (red), deterministic trajectory (green), best simulation obtained with the SLTM (black) and search area (contour plots for the 10,000 simulated trajectories) are shown.

Figure 6 (panels a, b, c and d) shows the separation distance between the actual and deterministic trajectory (black triangles) and between the actual and the best simulation trajectory obtained with the SLTM (grey circles) for the simulations shown in Figure 5 (panels a, b, c and d). As can be observed, the error decreases significantly with the SLTM approach. This separation distance was calculated for the entire simulations performed using the trajectory dataset presented in Figure 4 and Table 2. Results show that after 5 days of simulation the root mean square error was found to be 115 km and 37 km for the deterministic approach and the best simulation of the SLTM, respectively. Moreover, actual trajectories were in the areas where the model predicted that the drifter was likely to go through. These results show the capability of the SLTM for trajectory modelling.

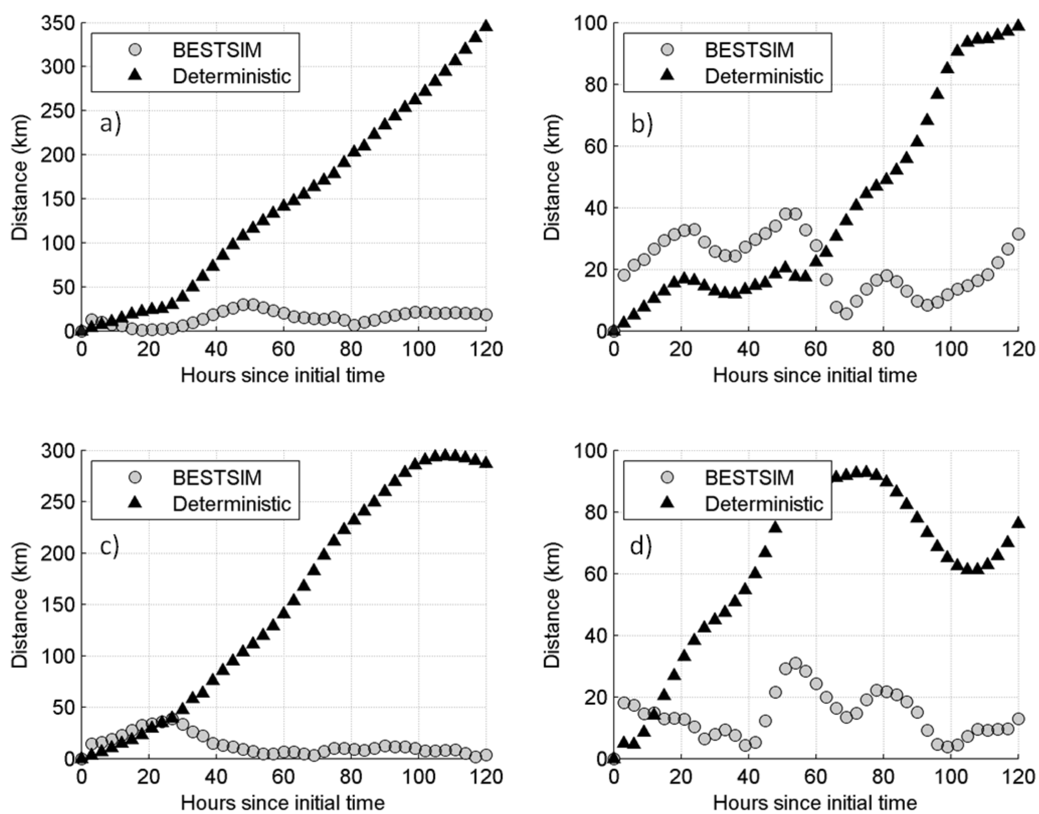

Figure 6 Example of the temporal evolution of the distance for the drifter #87806. The comparison between the actual drifter and the deterministic trajectory (black triangles) and the actual drifter and the best simulation obtained with the SLTM (grey circles) is shown.

5 Conclusions

This work presents the modelling of oil spills and drifter trajectories by means of a SLTM (Mínguez et al., 2012). This model is able to quantify the uncertainties in trajectory simulations and to define the most likely search area for possible trajectories. It is worth mentioning that once estimated the model parameters, the SLTM can be applied for forecasting oil spill trajectories. The model is used to simulate the trajectories of surface drifters deployed in the Gulf of Mexico during the *Deepwater Horizon* oil spill incident using surface currents provided by the Global HYCOM model. Note that the HYCOM is traditionally a deep ocean application model and, therefore, this work focuses on the simulation of the drifter trajectories located in deep ocean.

It is very important to point out that the proposed method does allow to quantify objectively how important are the uncertainties associated with both numerical modelling and model parameterization with respect to the possible trajectories. As a result, the method provides a search area where according to all uncertainties any oil spill particle could be located, and also defines the probability of that particle of being in that location. Thus the magnitude of the search area depends upon the quality of forcings, parameterization and numerical resolution. The method itself does not improve estimates but allows to know how good the model and data are. Any improvement of these aspects, especially about the forcings, will result in a reduction of uncertainty and, as a consequence, the search area will be narrower with the same probability of containing the actual trajectories. Nevertheless, the model is very useful to define the risks related to oil spill reaching particular areas of interest. This is very useful to mitigate the consequences of possible accidents and for the definition of emergency plans and prioritization of actions in coastal areas.

Finally, this work shows the capabilities of the presented stochastic methodology for oil spill and drifter trajectories modelling. Further studies are required to analyse the differences between drifter and oil spill trajectories. Moreover, further work is required to compare the stochastic simulations with the simulations provided by a typical oil spill Lagrangian model.

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